

LOCAL REFERENCE FRAMES AND QUANTUM SPACE-TIME

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Abstract

We argue that the account of Reference Frames quantum properties must change the standard space-time picture accepted in Quantum Mechanics. If RF is connected with some macroscopic solid object then its free quantum motion - wave packet smearing results in additional uncertainty into the measurement of test particle coordinate. It makes incorrect the use of Galilean or Lorentz space-time transformations between two RF and the special quantum space-time transformations are formulated. It results in generalized Klein- Gordon equation which depends on RF mass. Both space and time coordinates become the operators. In particular RF proper time becomes the operator depending of momentums spectra of this RF wave packet , from the point of view of other observer.

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1 Introduction

Some years ago and Kaufherr have shown that in nonrelativistic Quantum Mechanics (QM) the correct definition of physical reference frame (RF) must differ from commonly accepted one, which in fact was transferred copiously from Classical Physics [1]. The main reason is that in strict QM framework one should account the quantum properties not only of studied object, but also RF, despite the possible practical smallness. The most simple of this properties is the existence of Schrodinger wave packet of free macroscopic object with which RF is usually associated [2]. If this is the case it inevitably introduces additional uncertainty in the measurement of object space coordinate. Furthermore this effect account results in the coordinate transformations between two quantum RFs, principally different from the Galilean ones [1, 3].

Further studies of quantum RF effects can help to understand the aspects of Quantum space-time which now are extensively investigated [4]. The importance of RF quantum properties was noticed already in Quantum Gravity studies [5, 6]. In this paper we discuss mainly the construction of relativistically covariant quantum RF theory, by the analysis of some simple models. It will be shown that the state vector transformations between two RF obeys to relativistic invariance principles, but due to dependence on RF state vectors differs from Poincare Group transformations. The time ascribed to such RF becomes the operator. In particular proper time in each RF is the operator depending on its momentum, which introduces the quantum fluctuations in the classical Lorentz time boost in moving RF time measurements.

Our paper is organized as follows : in the rest of this chapter our model of quantum RF will be formulated and its compatibility with Quantum Measurement Theory discussed. In a chapter 2 the new canonical formalism of quantum RF states and their transformations developed, which is more simple and realistic then proposed in [1]. The relativistic equations for quantum RF and the resulting quantum space-time properties are regarded in chapter 3 . In a final chapter the obtained results and their interpretation are discussed.

In QM framework the system regarded as RF presumably should be able to measure the observables of studied quantum states and due to it to include measuring devices - detectors. As the realistic example of such RF we can regard the photoemulsion plate or the diamond crystal which can measure microparticle position relative to its c.m. and simultaneously record it. At first sight it seems that due to it quantum RF problem must use as its basis the detailed model of state vector collapse. Yet despite the multiple proposals up to now well established theory of collapse doesn't exist [7, 8]. Alternatively we'll show that our problem premises doesn't connected directly with the state vector collapse mechanism and as the result we can use two simple assumptions about the RF and detector states properties which are in the same time rather weak. The first one is that RF consists of finite number of atoms (usually rigidly connected) and have the finite mass. Our second assumption needs some preliminary comments. It's well known that the solution of Schrodinger equation for any free quantum system consisting of N constituents can

be presented as :

$$\Psi(\vec{r}_1, \dots, \vec{r}_n, t) = \sum c_l \Phi_l^c(\vec{R}_c, t) * \phi_l(\vec{r}_{i,j}, t) \quad (1)$$

where center of mass coordinate $\vec{R}_c = \sum m_i * \vec{r}_i / M$. $\vec{r}_{i,j} = \vec{r}_i - \vec{r}_j$ are the relative or 'internal' coordinates of constituents [2]. Here Φ_l^c describes the c.m. motion of the system. It demonstrates that the evolution of the system is separated into the external evolution of pointlike particle M and the internal evolution completely defined by $\phi_l(\vec{r}_{i,j}, t)$. So the internal evolution is independent of whether the system is localized in the standard macroscopic 'absolute' reference frame (ARF) or not. Relativistic QM and Field Theory evidences that the factorization of c.m. and relative motion holds true even for nonpotential forces and variable N in the secondarily quantized systems [12]. Moreover this factorization expected to be correct for nonrelativistic systems where binding energy is much less than its mass m_1 , which is characteristic for the most of real detectors. Consequently it's reasonable to extend this result on the detector states despite we don't know their exact structure. We'll use it quite restrictively and assume that the factorization of the c.m. motion holds for RF only in the time interval T from RF preparation moment, until the act of measurement starts, i.e. when the measured particle collides with it. More exactly our second and last assumption about observer properties is that during period T its state is described by wave function generalizing (1) of the form

$$\Psi(R_c, q, t) = \sum c_l \Phi_l^c(R_c, t) * \phi_l(q, t)$$

where q denote all internal detector degrees of freedom which evolve during T according to Schrodinger equation (or some field equation). Its possible violation at later time when particle state collapse occurs is unimportant for our model.

To simplify our calculations we'll take below all $c_l = 0$ except c_1 which wouldn't influence our final results. The common opinion is that to observe experimentally measurable smearing of macroscopic object demands too large time, but for some mesoscopic experiments it can be reasonably small to be tested in the laboratory conditions [11]. We don't consider in our study the influence of RF recoil effects on the measurements results which can be made arbitrarily small [1].

2 Quantum Coordinates Transformations

We remind here only briefly the physical meaning of quantum RF, because in pioneering paper [1] authors analyzed in detail gedankenexperiments with quantum RF and interested reader can find the discussion there. Suppose that in absolute RF (in two dimensions x, y) the wave packet of RF F^1 is described by wave function $\psi_1(x)\phi_1(y)$. The test particle n with mass $m_n = m_2$ belongs to narrow beam which average velocity is orthogonal to x axe and its wave function can be presented as $\psi_n(x)\phi_n(y)$. We want to find n wave function for the observer situated in F^1 rest frame. Formally it can be done by means of canonical transformations described in detail below. But from the qualitative side it follows that in the simplest case when

n beam is well localized and $\psi_n(x)$ can be approximated by delta-function $\delta(x - x_b)$ this wave function in F^1 (its x-component) $\psi'_n(x_n) = \psi_1(x_n - x_b)$. It shows that if F^1 wave packet have average width σ_x then from the 'point of view' of observer in F^1 each object localized in ARF acquires wave packet of the same width.

Let's assume that F^1 includes detector D_0 which can measure the distance between n and F^1 c.m. Then considering the collapse induced by n, D_0 interaction F^1 and ARF observers will treat the same event unambiguously as n detection (or its flight through D_0). In observer reference frame F^1 it reveals itself by the detection and amplification process in D_0 initiated by n absorption. For ARF the collapse results from the nonobservation of neutron in a due time - so called negative result experiment. So the signal in F^1 will have the same relative probability as in ARF. This measurement means not only the reduction of ψ'_n in F^1 , but also the reduction of ψ_1 in ARF. Due to it we'll assume always that all measurements are performed on quantum ensemble of observers F^1 . It means that each event is resulted from the interaction between the 'fresh' RF and particle, prepared both in the specified quantum states, alike the particle alone in the standard experiment. As we have no reason to assume that the transition from ARF to F^1 can transfer pure states to mixed ones we must conclude that this distribution is defined by neutron wave function in F^1 . It means that the result of measurement in F^1 is also described by QM Reduction postulate, i.e. that initial state during the measurement by RF detector evolve into the mixture of the measured observable eigenstates.

After this qualitative example we'll regard the general nonrelativistic formalism, which differ from described in [1]. Consider the system S_N of N objects B^i which include N_g pointlike 'particles' G^i and N_f frames F^i , which in principle can have also some internal degrees of freedom described by (1). For the start we'll assume that particles and RF coordinates \vec{r}_i are given in absolute (classical) ARF having very large mass m_A and formally having coordinate $\vec{r}_A = 0$ and momentum $\vec{p}_A = 0$. At the later stage of the study this assumption can be abandoned and only relations between quantum RF regarded. We should find two transformation operators - from ARF to quantum RF, and between two quantum RF, but it'll be shown that in most general approach they coincide. We'll use Jacoby canonical coordinates $\vec{q}_j^l, 1 \leq j \leq N$, which for F^l are equal :

$$\vec{q}_i^l = \frac{\sum_{j=i+1}^N m_j^l \vec{r}_j^l}{M_N^{l,i+1}} - \vec{r}_i^l, \quad \vec{q}_N^l = \vec{R}_{cm} - \vec{r}_A \quad (2)$$

Here $\vec{r}_j^1 = \vec{r}_j, m_j^1 = m_j$, and for $l > 1$:

$$\vec{r}_j^l = \vec{r}_j, \quad j > l, \quad \vec{r}_j^l = \vec{r}_{j-1}^l, \quad 1 \leq j \leq l, \quad \vec{r}_1^l = \vec{r}_l$$

The same relations connect for m_j^l and m_j . $M_n^{l,i} = \sum_{j=i}^n m_j^l$ (if upper index i, l are omitted, it assumed that $i, l = 1$) Conjugated to q_i^1 ($i = 1, N$) canonical momentums are :

$$\vec{\pi}_i^1 = \mu'_i \left(\frac{\vec{p}_{i+1}^s}{M_N^{i+1}} - \frac{\vec{p}_i}{m_i} \right), \quad \vec{\pi}_N^1 = \vec{p}_1^s \quad (3)$$

where $\vec{p}_i^s = \sum_{j=i}^N \vec{p}_j$, and reduced mass $\mu_i'^{-1} = (M_N^{i+1})^{-1} + m_i^{-1}$.

The relative coordinates $\vec{r}_j - \vec{r}_1$ can be represented as the linear sum of several coordinates \vec{q}_i^1 ; they don't constitute canonical set due to quantum motion of F^1 .

We consider first the transformation between two quantum RF and start from the simplest case $N_f = 2, N_g = 0$. This is just the space reflection of F^1 coordinate $\vec{q}_1^2 = -\vec{q}_1^1$ performed by the parity operator \hat{P}_1 . The next case $N_f = 2, N_g = 1$ is \vec{q}^1 coordinates bilinear transformation exchanging \vec{r}_2, \vec{r}_1 :

$$q_{1,2}^2 = \hat{U}_{2,1} q^1 \hat{U}_{2,1}^+ = a_{1,2} q_1^1 + b_{1,2} q_2^1 \quad (4)$$

Corresponding unitary operator can be decomposed as $\hat{U} = \hat{C}_2 \hat{R} \hat{C}_1$, where $\hat{C}_{1,2}$ are the dilatation operators, which action changes the coordinate scale. For example \hat{C}_1 results in $\vec{q}_i^1 = c_1^i \vec{q}_i^1$, where $c_{1,2}^i$ proportional to μ_i' .

\hat{R} is the rotation on $\vec{q}_{1,2}^1$ intermediate coordinates hypersurface on the angle:

$$\beta = -\arccos\left[\frac{m_2 m_1}{(m_3 + m_2)(m_1 + m_3)}\right]^{\frac{1}{2}}$$

For the general case $N > 3$ it's possible nonetheless to decompose the transformation from F^j to F^k as the product of analogous bilinear operators. Really if to denote as $\hat{S}_{i+1,i}$ the operator exchanging F^i, F^{i+1} in \vec{q}^1 set, As follows from (2) it changes in fact only $\vec{q}_i^1, \vec{q}_{i+1}^1$ pair. $\hat{U}_{2,1} = \hat{S}_{2,1}$ and all $\hat{S}_{j,j-1}$ have the analogous form, changing only parameters β, c_k^i . Then the transformation operator from F^1 to F^k is:

$$\hat{U}_{k,1} = \hat{S}_{2,1} \hat{S}_{3,2} \dots \hat{S}_{k,k-1} \quad (5)$$

It follows immediately that the transformation from F^j to F^k is $\hat{U}_{j,k} = \hat{U}_{k,1} \hat{U}_{j,1}^{-1}$.

To find the transformation operator from the classical ARF to F^1 we'll regard ARF as the quantum object B^{N+1} with infinite m_{N+1} belonging to extended system S_{N+1} . ARF 'classical' set is $\vec{q}_i^A = \vec{r}_i - \vec{r}_A$, but acting by parity operators we'll transform it to $\vec{q}_i^A = -\vec{q}_i^A$. Then it's easy to see that for S_{N+1} $\vec{q}_i^1 = \vec{q}_i^A$ as follows from (2). Note that formally we can regard also each particle G^j as RF and perform for them the transformations $\hat{S}_{j+1,j}$ described above. Then omitting simple calculations we obtain that operator performing transformations from ARF to F^1 is equal to $\hat{U}_{A,1} = \hat{U}_{N+1,1}$ for infinite m_{N+1} . In this case new \vec{q}^1 set for S_{N+1} can be rewritten as the function of $\vec{r}_i^1, \vec{r}_{N+1}^1 = \vec{r}_A^1$ of (2) to which formally must be added $\vec{q}_{N+1}^1 = \vec{r}_A - \vec{r}_E$, where E is some other classical RF.

The free Hamiltonian of the system objects motion in ARF is:

$$\hat{H} = \hat{H}_s + \hat{H}_c = \frac{(\vec{\pi}_N^1)^2}{2M_N} + \sum_{j=1}^{N-1} \frac{(\vec{\pi}_j^1)^2}{2\mu_j'} \quad (6)$$

Hamiltonian of S_N in F^1 should depend on relative B^i momentums only, so we can regard \hat{H}_c as the candidate for its role. Yet relativistic analysis given below introduces some corrections to \hat{H}_c . Note that even settling $\vec{r}_1^1 = 0, \vec{p}_1^1 = 0$ for F^1

we must account them as the operators in commutation relations ,as was stressed in [9].

This transformations becomes more complicated if we take into account the quantum rotation of our RF relative to ARF, which introduce additional angular uncertainty into objects coordinates [1]. We'll consider here only 2-dimensional rotations, for which this uncertainty is connected with F^1 orientation relative to ARF axes. If F^1 is the solid object its orientation relative to ARF can be extracted from the relative (internal) coordinates of F^1 constituents (atoms). For the simplicity we assume that F^1 have the dipole structure and all its mass concentrated around 2 points $\vec{r}_{a1}, \vec{r}_{b1}$ so that this F^1 internal coordinate is $\vec{r}_{a1} - \vec{r}_{b1}$ or in polar coordinates r_1^d, θ_1 . Note that r_1^d is observable which eigenvalue defined by F^1 constituents interaction. Even if F^1 have some complex form its orientation defined by the same single observable θ_1 and only rotational effective mass m_1^d will depend on this. Thus after performing coordinate transformation $\hat{U}_{A,1}$ from ARF to F^1 c.m. we'll rotate all the objects (including ARF) around it on the uncertain angle θ_1 , so the complete transformation is $\hat{U}_{A,1}^T = \hat{U}_{A,1}^R \hat{U}_{A,1}$. In its turn this rotation operator can be decomposed as $\hat{U}_{A,1}^R = \hat{U}_{A,1}^c \hat{U}_{A,1}^d$, representing the rotation of objects c.m coordinates \vec{q}_i^A and F^i constituents coordinates. Their calculations are in fact straightforward and follows directly from the properties of standard orbital momentum operator so we omit the details. θ_1 is independent of F^1 c.m. coordinate \vec{r}_1 and due to it the transformation of \vec{q}_i^A is performed analogously to rotation on fixed angle. Their transformation operator is:

$$\hat{U}_{A,1}^c = e^{-i\theta_1 L_z}, \quad L_z = \sum_{i=1}^N l_{zi}, \quad l_{zi} = -id/d\alpha_i \quad (7)$$

where α_i is the polar angle coordinate of \vec{q}_i^A . It results in coordinate transformation :

$$\begin{aligned} q_{xi}^{1r} &= q_{xi}^1 \cos \theta_1 + q_{yi}^1 \sin \theta_1 \\ q_{yi}^{1r} &= -q_{xi}^1 \sin \theta_1 + q_{yi}^1 \cos \theta_1 \end{aligned} \quad (8)$$

So the new polar angle is $\alpha_i^r = \alpha_i - \theta_1$. The transformation of canonical momentums is analogous :

$$\begin{aligned} \pi_{xi}^{1r} &= \pi_{xi}^1 \cos \theta_1 + \pi_{yi}^1 \sin \theta_1 \\ \pi_{yi}^{1,r} &= -\pi_{xi}^1 \sin \theta_1 + \pi_{yi}^1 \cos \theta_1 \end{aligned} \quad (9)$$

As can be easily checked Hamiltonian \hat{H}_c of (6) is invariant under this transformation.

The rotation of F^1 is performed by the operator $\hat{U}_{A,1}^d$, which action settles θ_1 to zero ,and introduce in place of it the new observable θ_1^r which corresponds to ARF angle in F^1 . If to denote θ_j - orientation angles for F^j constituents and $l_j^d = -i\frac{\partial}{\partial \theta_j}$ their orbital momentums , then the transformation operator is :

$$\hat{U}_{A,1}^d = \hat{P}_1^d \exp(i\theta_1 L_d), \quad L_d = \sum_{i=2}^{N_f} l_i^d \quad (10)$$

It results in new canonical observables :

$$\begin{aligned}\theta_j^r &= \theta_j - \theta_1, & l_j'^d &= l_j^d \quad j \neq 1 \\ \theta_1^r &= \theta_A^r = -\theta_1, & l_1'^d &= l_A^d = -l_1^d - L_d\end{aligned}\tag{11}$$

where \hat{P}_1^d - parity operator for θ_1 . The new coordinates can be interpreted as corresponding to F^1 dipole rest frame , where its own angle θ_1' is fixed to zero but ARF angle in F^1 θ_1^r becomes uncertain and formally ARF acquires the orbital momentum l_A^d .

Analogous considerations permit to find rotational transformation $\hat{U}_{i,1}^R$ from F^1 to F^i . It means the additional rotation of all the objects on the angle $\theta_i^r = \theta_i - \theta_1$. Consequently the form of \hat{U}^c operator for it is conserved and it's just necessary to substitute θ_i^r in it as parameter. Operator $\hat{U}_{i,1}^d$ can be expressed as :

$$\hat{U}_{i,1}^d = \hat{U}_{A,i}^d (\hat{U}_{A,1}^d)^{-1}\tag{12}$$

where $\hat{U}_{A,i}^d$ can be easily found from (10).

Assuming that any F^i have analogous to F^1 dipole form the part of constituents relative motion Hamiltonian which depends on orientation in ARF :

$$\hat{H}_i = \sum_{j=1}^{N_f} \frac{1}{m_j^d r_j^{d2}} \frac{\partial^2}{\partial \theta_j^2}\tag{13}$$

where m_j^d is the effective mass of the rotational moment which for the dipole is equal to its reduced mass. It follows that this operator is invariant of $\hat{U}_{A,1}^d$ transformations.

So we get the conclusive and noncontroversial description of G^i and F^i coordinate transformation to F^1 defined by Hamiltonian $\hat{H}_c + \hat{H}_i$. Yet this transformation can result in the change of the objects G^i wave functions Ψ^1 in F^1 which will depend on F^1 orbital momentum which should be accounted performing the initial functions transformation.

For $d = 3$ the mathematical calculations are analogous , but more tedious , if to account that any rotation in space can be decomposed as three consequent rotations in the specified mutually orthogonal planes. We omit this calculations here, and just explain new features appearing. To describe this rotation F^1 should have the necessary geometric structure , the simplest of which is the triangle abc with masses concentrated in its vertexes. Then Z' axe can be chosen to be orthogonal to the triangle plane and X' directed along ab side. Then the transformation which aligns ARF and F^1 axes can be performed rotating consequently ARF around X', Y', Z' on the uncertain angles $\theta_x, \theta_y, \theta_z$. Each of three operators performing it is the analog of $\hat{U}_{A,1}^R$ described above.

Jacoby formalism described here is more simple then the formalism of [1] and can be easily developed for relativistic quantum RF description.

3 Relativistic Equations

The relativistic covariant formalism of quantum RF will be studied with the model of relativistic wave packets of macroscopic objects regarded as quantum RF . It supposed that in RF all its constituents spins and orbital momentums are roughly compensated and can be neglected. For the simplicity we'll neglect quantum rotation effects, described in the former chapter.

In nonrelativistic mechanics time t is universal and is independent of observer. In relativistic case each observer in principle has its own proper time τ measured by his clocks, which is presented in evolution equation for any object in this RF . We don't know yet the origin of the physical time , but phenomenologically we can associate it with the clock hands motion, or more exactly with the measurement and recording of their current position by observer [17]. This motion is stipulated by some irreversible processes which are practically unstudied on quantum level. It seems there is a strong and deep analogy between irreversible wave function collapse in the measurement and clock hands motion+measurement which can be regarded as the system self-measurement [10]. Meanwhile without choosing one or other mechanism , it's possible to assume that as in the case of the position measurement this internal processes can be disentangled from the clocks c.m. motion. Then clocks + observer F^2 wave packet evolution can be described by the relativistic equation for their c.m. motion relative to RF F^1 and F^2 internal degrees of freedom evolution which define its proper time τ_2 supposedly are factorized from it. In this packet different momentums and consequently velocities relative to external observer F^1 are presented. It makes impossible to connect external time τ_1 and F^2 proper time τ_2 by any Lorentz transformation, characterized by unique definite Lorentz factor $\gamma(\vec{v})$ [14].

To illustrate the main idea we remind the well-known situation with the relativistic lifetime dilation of unstable particles or metastable atoms. Imagine that the prepared beam of them is the superposition of two or more momentums eigenstates having different Lorentz factors γ_i . Then detecting their decay products we'll find the superposition of several lifetime exponents , resulting from the fact that for each beam component Lorentz time boost has its own value. If in some sense this unstable state can be regarded as elementary clock when their time rate for the external observer is defined by the superposition of Lorentz boosts responding to this momentums.

This arguments suppose that the proper time of any quantum RF being the parameter in his rest frame simultaneously will be the operator for other quantum RF. If this is the case the proper time τ_2 of F^2 in F^1 can be the parameter depending operator, where parameter is τ_1 .

$$\hat{\tau}_2 = \hat{F}(\tau_1) = \hat{B}_{12}\tau_1$$

,where \hat{B}_{12} can be called Lorentz boost operator,which can be the function of F^1, F^2 relative momentum. We regard F^1 proper time as parameter, which means that the possible quantum fluctuations of F^1 clocks are supposed to be small [17].

To define \hat{B}_{12} it's necessary to find F^2 Hamiltonian for which we consider first the relativistic motion of free spinless particle G^2 in F^1 . Obviously in relativistic case Hamiltonian of relative motion of very heavy RF and light particle should approximate Klein-Gordon square root Hamiltonian, but in general it can differ from it [12]. The main idea how to find it is the same as in nonrelativistic case : to separate the system c. m. motion and the relative motion of the system parts [15].

We'll consider first the evolution of system S_2 of RF F^1 and the particle G^2 which momentums \vec{p}_i are defined in some classical ARF. If to regard initially prepared states including only positive energy components (not considering antiparticles at this stage) , then their joint state vector evolution in ARF is described by square root Hamiltonian [12]:

$$-i\frac{d\psi^0}{d\tau_0} = [(m_1^2 + \vec{p}_1^2)^{\frac{1}{2}} + (m_2^2 + \vec{p}_2^2)^{\frac{1}{2}}]\psi^0 \quad (14)$$

in momentum representation. To obtain from it the Hamiltonian of G^2 in F^1 we remind that the objects relative motion is characterized by their invariant mass s^m , which is equal to system total energy in its c.m.s.. In our case it's equal to :

$$s_{12}^m = (m_1^2 + \vec{q}_{12}^2)^{\frac{1}{2}} + (m_2^2 + \vec{q}_{12}^2)^{\frac{1}{2}} \quad (15)$$

,where \vec{q}_{12} is G^2 momentum in c.m.s. If to define S_2 total momentum in its c.m.s. p_μ^s , which 4-th component is s_{12}^m , then transforming it to F^1 rest frame one obtains 4-vector p_μ^1 which 4-component is equal :

$$E^1 = [(s_{12}^m)^2 + \vec{p}_{12}^2]^{\frac{1}{2}} = m_1 + (m_2^2 + \vec{p}_{12}^2)^{\frac{1}{2}} \quad (16)$$

Here \vec{p}_{12} is classical G^2 momentum in F^1 rest frame

$$\vec{p}_{12} = \frac{s_{12}^m \vec{q}_{12}}{m_1} = \frac{E_1 \vec{p}_2 - E_2 \vec{p}_1}{m_1} \quad (17)$$

where E_1, E_2 are the energies of F^1, G^2 in ARF. From this classical calculations and Correspondence principle (which in relativistic QM should be applied carefully) we can regard E^1 as possible form of Hamiltonian \hat{H}^1 in F^1 rest frame and the evolution equation for G_2 for corresponding proper time τ_1 is :

$$\hat{H}^1 \psi^1(\vec{p}_{12}, \tau_1) = -i \frac{d\psi^1(\vec{p}_{12}, \tau_1)}{d\tau_1} \quad (18)$$

It's easy to note that \hat{H}^1 depends only of relative motion observables and in particular can be rewritten as function of \vec{q}_{12} . This equation will coincide with Klein-Gordon one, if we consider m_1 as arbitrary constant added to G^2 energy. Consequently we can use in F^1 the same momentum eigenstates spectral decomposition and the states scalar product [12].

Space coordinate operator in F^1 is difficult to define unambiguously, as usual in relativistic QM, but Newton-Wigner ansatz can be used without complications [13].

$$x_{12} = i \frac{d}{dp_{x,12}} - i \frac{p_{x,12}}{2(E^1 - m_1)^2} \quad (19)$$

In this framework F^2 proper time operator $\hat{\tau}_2$ in F^1 can be found from the correspondence with the classical Lorentz time boost appearing in moving clock time measurement $\Delta t_2 = \gamma_2^{-1} \Delta t_1$, where γ_2 is F^2 Lorentz factor [14]. Rewriting it as function of momentums in F^1 one obtains :

$$\hat{\tau}_2 = (\vec{p}_{12}^2 + m_2^2)^{-\frac{1}{2}} m_2 \tau_1 \quad (20)$$

Due to its dependence only on τ_1 - parameter and momentums this operator is self-adjoint and doesn't suppose the use of POV measures, used in some models for time operator [16]. Note that the operators $\hat{x}_{12}, \hat{\tau}_2$ doesn't commute, due to $\hat{\tau}_2$ dependence on $\hat{p}_{x,12}$. This result is analogous to Noncommutative Geometry and Quantum Groups predictions for quantum space-time at Plank scale, yet our scale is much larger [4]. Note that this approach is completely symmetrical and the operator analogous to (20) relates the time $\hat{\tau}_1$ in F^1 and F^2 proper time - parameter τ_2 . By himself (or itself) F^2 can't find any consequences of time arrows superposition registrated by external F^1 , because for F^2 exists only unique proper time τ_2 . The new effect will be found only when F^1 and F^2 will compare their initially synchronized clocks.

Analogously to Classical Relativity average time boost depends on whether F^1 measures F^2 observables, as we considered or vice versa. To perform this measurement we must have at least two synchronized objects with clocks F_a^1 and F_b^1 , which make two F^1 and F^2 nonequivalent. If this experiment will be repeated several times (to perform quantum ensemble) it'll reveal not only classical Lorentz time boost, but also the statistical spread having quantum origin and proportional to F^1 time interval Δt_1 and F^2 momentum spread.

If the number of objects $N > 2$ the modified clasterization formalism can be used, which will be described here for the case $N = 3$ [15]. According to previous arguments Hamiltonian in F^1 describing the two particles G^2, G^3 state evolution for proper time τ_1 is equal to sum of two single particles energies. Rewritten through the invariant system observables it have the form :

$$\hat{H}^1 = m_1 + [(s_{23}^m)^2 + \vec{p}_{23}^2]^{\frac{1}{2}} \quad (21)$$

, where s_{23}^m is two particles G^2, G^3 invariant mass, which dependence on \vec{q}_{23} is analogous to (15). In clasterization formalism at first level we consider the relative motion of G^2, G^3 defined by \vec{q}_{23} . At second level we regard them as the single quasiparticle - cluster C_{23} with mass s_{23}^m and \vec{p}_{23} momentum in F^1 . So at any level we regard the relative motion of two objects only. Small Hilbert space H_{23} with the basis $|\vec{q}_{23}\rangle$ can be extracted from the total space H_s^1 , which properties are the same as for classic F^1 with infinite mass [15]. This clasterization procedure can be extended in the obvious inductive way to incorporate an arbitrary number of the objects.

Due to appearance of quantum proper time the transformation operator between two quantum RFs $\hat{U}_{21}(\tau_2, \tau_1)$ is quite intricated, and to obtain it general form needs further studies. To illustrate the physical meaning of this transformation, we'll discuss briefly the transformation of single particle G^3 state between F^1 and F^2 . We'll regard the particular case for which in F^1 at time $\tau_1^0 = 0$ the joint state

vector of F^2 and m_3 - is $\psi_{in}^1(\vec{p}_{23}, \vec{q}_{23}) = \sum c_{jk}^1 |\vec{p}_{23,j}\rangle |\vec{q}_{23,k}\rangle$ is given, where \vec{p}_{23} is F^2, G^3 total momentum in F^1 . F^2 proper time τ_2 is synchronized with F^1 at this moment $\tau_2^0 = \tau_1^0$. Due to unambiguous correspondence between the $\vec{p}_{13}, \vec{q}_{13}$ and $\vec{p}_{23}, \vec{q}_{23}$ phase space points we can assume that the state vector $\psi_{in}^2(\vec{p}_{13}, \vec{q}_{13})$ in F^2 is obtained acting on ψ_{in}^1 by some unitary operator \hat{V}_{21} . Time evolution operator in F^1 $\hat{W}_1(\tau_1) = \exp(i\tau_1 \hat{H}^1)$ is defined by Hamiltonian \hat{H}^1 only, and the same relation is true for $\hat{W}_2(\tau_2)$ in F^2 . Then G^3 state in F^2 at any τ_2 can be obtained by the action of operator $\hat{W}_2(\tau_2) \hat{V}_{21} \hat{W}_1^{-1}(\tau_1)$ on the corresponding G^3 state in F^1 . It means that despite τ_2 and τ_1 are correlated only statistically and have some quantum fluctuations, G^3 state vectors in F^2, F^1 at this moments are related unambiguously.

Now we'll consider obtained results in nonrelativistic limit. It's easy to see that in the limit $\vec{p}_{12} \rightarrow 0$ Hamiltonian (16) differs from \hat{H}_c of (6) by the factor $k_m = \frac{m_1+m_2}{m_1}$. It results from energy-momentum Lorentz transformation from c.m.s. to F^1 . We've chosen Newton-Wigner space coordinate operator in F^1 x_{12} , which is canonical conjugated to \vec{p}_{12} . In nonrelativistic limit it's equal to $x_{21} = k_m^{-1}(x_2 - x_1)$, where x_1, x_2 are coordinates in ARF. This result doesn't broke transformation invariance, because there is no established length scale in QM.

To illustrate our approach to quantum time we'll regard the simple model of the quantum clocks and RF in which F^1 includes some ensemble (for example the crystal) of β -radioactive atoms [17]. Their nucleus can radiate neutrino ν (together with the electron partner) which due to its very small cross-section practically can't be reflected by any mirror and reabsorbed by this nucleus to restore the initial state. Then for our purposes this decay can be regarded as the irreversible stochastic process. Taking the trace over ν degrees of freedom, the final nucleus state can be described by the density matrix of mixed state $\rho_N(t)$ and the proper time of this clocks of F^1 can be defined as:

$$\tau_1 = -T_d \ln(1 - \frac{N_d}{N_0})$$

where N_0 is the initial number of this atoms N_d - the number of decays, T_d is the nucleus lifetime. The corrections to exponential decays, appearing at time much larger then lifetime is neglected [18]. It's easy to understand from the previous discussion how the superposition of Lorentz boosts can be applied to such system state, if its state vector has momentum spread.

We consider in fact infrared limit for macroscopic object, so the role of negative energy states, which is important for the standard relativistic problems must be small.

4 Concluding Remarks

We've shown that the extrapolation of QM laws on free macroscopic objects demands to change the approach to the space-time coordinate frames which was taken copiously from Classical Physics. It seems that QM permits the existence of RF manifold, the transformations between which principally can't be reduced to Galilean or Lorentz transformations. It means that observer can't measure its own spread

in space, so as follows from Mach Principle it doesn't exist. The physical meaning have only the spread of relative coordinates of RF and some external object which can be measured by this RF or other observer.

Historically QM formulation started from defining the wave functions on Euclidean 3-space R^3 which constitute Hilbert space H_s . In the alternative approach accepted here we can regard H_s as primordial states manifold. Introducing particular Hamiltonian results in the relative asymmetry of H_s vectors which permit to define R^3 as a spectrum of the continuous observable \hat{r} which eigenstates are $|\vec{r}_i\rangle$. But as we've shown for several quantum objects one of which is RF this definition become ambiguous and have several alternative solutions defining R^3 on H_s . In the relativistic case the situation is more complicated, yet as we've shown it results in ambiguous Minkovsky space-time definition.

In our work we demanded strictly that each RF must be quantum observer i.e. to be able to measure state vector parameters. But we should understand whether this ability is main property characterizing RF ? In classical Physics this ability doesn't influence the system principal dynamical properties. In QM at first sight we can't claim it true or false finally because we don't have the established theory of collapse. But it can be seen from our analysis that collapse is needed in any RF only to measure the wave functions parameters at some t . Alternatively this parameters at any RF can be calculated given the initial experimental conditions without performing the additional measurements. It's quite reasonable to take that quantum states have objective meaning and exist independently of their measurability by the particular observer, so this ability probably can't be decisive for this problem. It means that we can connect RF with the system which doesn't include detectors, which can weaken and simplify our assumptions about RF. We can assume that primordial for RF is the ability, which complex solid states have, to reproduce and record the space and time points ordering with which objects wave functions are related.

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